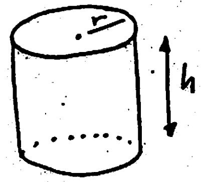
Problèmes mini-max

Exercice 9

$$V = \pi r^2 h = 8\,m^3 \Rightarrow h = \frac{8}{\pi r^2}$$



a) $A = 2\pi r \cdot h + \pi r^2 = 2\pi r \cdot \frac{8}{\pi r^2} + \pi r^2 = \frac{16}{r} + \pi r^2$

Conditions : $A' = 0$ et $A'' > 0$

$$A' = -\frac{16}{r^2} + 2\pi r \longrightarrow A' = 0 \iff \frac{16}{r^2} = 2\pi r \quad \Big|\cdot r^2$$

$$r^3 = \frac{16}{2\pi} \Rightarrow \underline{r = \frac{2}{\sqrt[3]{\pi}} \cong 1{,}3656\,m}$$

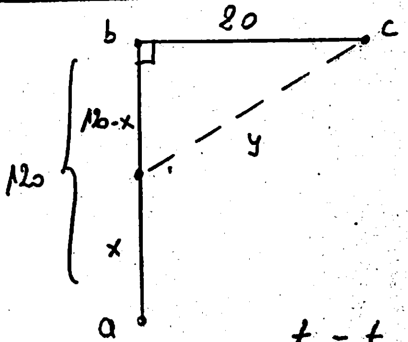
$$A'' = \frac{32}{r^3} + 2\pi > 0 \quad \text{si } r = \frac{2}{\sqrt[3]{\pi}}$$

si $r = \frac{2}{\sqrt[3]{\pi}}$, alors $h = \frac{2}{\sqrt[3]{\pi}}$, donc $r = h$

b) $A = 2\pi r h + 2 \cdot \pi r^2$

$A' = -\frac{16}{r^2} + 4\pi r \qquad A' = 0 \iff r^3 = \frac{4}{\pi} \Rightarrow \underline{r = \sqrt[3]{\frac{4}{\pi}} \text{ et } h = 2 \cdot \sqrt[3]{\frac{4}{\pi}}}$

Exercice 10



$$V_t = 150\,km/h \quad \text{et} \quad V_v = 90\,km/h$$

$$t_t = \frac{x}{150} \quad \text{et} \quad t_v = \frac{y}{90}$$

$$y^2 = 80^2 + (120 - x)^2 = x^2 - 240x + 20800$$

$$y = \sqrt{x^2 - 240x + 20800}$$

$$t = t_t + t_v = \frac{x}{150} + \frac{\sqrt{x^2 - 240x + 20800}}{90}$$

$$t' = \frac{1}{150} + \frac{x - 120}{90\sqrt{x^2 - 240x + 20800}}$$

$$t' = 0 \iff x - 120 = -\frac{90\sqrt{\quad}}{150}$$

$$5x - 600 = -3\sqrt{x^2 - 240x + 20800} \quad \Big|^2$$

$$25x^2 - 6000x + 360000 = 9(x^2 - 240x + 20800)$$

$$x^2 - 240x + 10800 = 0$$

$$\Delta = 14400 = 120^2$$

$x_1 = \frac{240 + 120}{2} = 180\,km$ à écarter, car $x < 120\,km$

$x_2 = \frac{240 - 120}{2} = \underline{\underline{60\,km}}$ (trajet total : $x + y = 160\,km$)

$(t'' > 0)$

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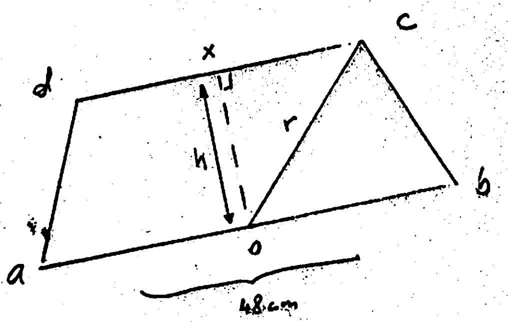
Exercice 11



$|oa| = |ob| = |oc| = r = 24\text{ cm}$

A = base moyenne · hauteur

$$A = \frac{48+x}{2} \cdot h$$

Calcul de h : $r^2 = \left(\frac{x}{2}\right)^2 + h^2$

$$h^2 = 24^2 - \frac{x^2}{4}$$

$$h = \frac{\sqrt{2304-x^2}}{2}$$

$$A = \frac{48+x}{2} \cdot \frac{\sqrt{2304-x^2}}{2} = \frac{(48+x)\left(2304-x^2\right)^{\frac{1}{2}}}{4}$$

$$A' = \frac{1 \cdot \sqrt{2304-x^2}}{4} + \frac{48+x}{4} \cdot \frac{-2x}{2\sqrt{2304-x^2}}$$

$$= \frac{\sqrt{2304-x^2}}{4} - \frac{(48+x)x}{4\sqrt{2304-x^2}} = \frac{(2304-x^2) - x(48+x)}{4\sqrt{2304-x^2}}$$

$$A' = \frac{-x^2-24x+1152}{2\sqrt{2304-x^2}}$$

$$A' = 0 \Rightarrow x^2 + 24x - 1152 = 0 \qquad \Delta = 5184$$

$$x_1 = \frac{24-72}{-2} = \underline{\underline{24\text{ cm}}}$$

$$x_2 = \frac{24+72}{-2} = -48\text{ cm} \quad \text{à écarter}$$

$$A'' = \frac{2\sqrt{}\,(-2x-24) - (-x^2-24x+1152) \cdot 2 \cdot \frac{1}{2} \, \frac{-2x}{\sqrt{}}}{4(-x^2-24x+1152)}$$

$$= \frac{2\sqrt{}\,(-2x-24) + \frac{2x(-x^2-24x+1152)}{\sqrt{}}}{4(-x^2-24x+1152)}$$

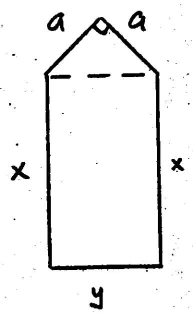
$$= \frac{\frac{2(-x^2-24x+1152)(-2x-24) + 2x(-x^2-24x+1152)}{\sqrt{}}}{4(-x^2-24x+1152)}$$

$$= \frac{2\left[(\quad)(-2x-24) + x(\quad)\right]}{4\sqrt{}\,(\quad)} = \frac{(\quad)(-x-24)}{2\sqrt{}\,(\quad)}$$

$$= \frac{-x-24}{2\sqrt{-x^2-24x+1152}} \quad \underline{\underline{< 0}} \quad \text{si } x = 24\text{ cm}$$

Problèmes mini-max

Exercice 12



périmètre: $2a + 2x + y = 32{,}485$ m

Δ rectangle : $y^2 = a^2 + a^2 = 2a^2$

$a^2 = \frac{y^2}{2} \Rightarrow a = \frac{\sqrt{2}}{2} \cdot y$

$p : \cancel{2}y \frac{\sqrt{2}}{\cancel{2}} + 2x + y = 32{,}485$ m

$2x + y(1+\sqrt{2}) = 32{,}485$ m

$x = \frac{32{,}485 - y(1+\sqrt{2})}{2}$

$A_{\square} = xy \quad , A_{\Delta} = \frac{a \cdot a}{2} = \frac{y^2}{4}$

$A = xy + \frac{y^2}{4}$

$= (16{,}2425 - \frac{1+\sqrt{2}}{2} y) y + \frac{y^2}{4}$

$A' = 16{,}2425 - \frac{1+2\sqrt{2}}{2} y$

$A' = 0 \Rightarrow y = 16{,}2425 \cdot \frac{2}{1+2\sqrt{2}} = \underline{\underline{8{,}485 \text{ m}}}$

$\underline{\underline{x = 6 \text{ m}}}$

$A'' = -\frac{1+2\sqrt{2}}{2} < 0$

Exercice 13

a) $V = \pi r^2 \cdot h = 100 \text{ cm}^3 \Rightarrow h = \frac{100}{\pi r^2}$

$A = 2\pi r \cdot h + 2\pi r^2 = \frac{200}{r} + 2\pi r^2$

$A' = -\frac{200}{r^2} + 4\pi r$

$A' = 0 \Leftrightarrow \frac{200}{r^2} = 4\pi r \quad | \cdot r^2$

$r = \sqrt[3]{\frac{50}{\pi}} \cong 2{,}52$ cm $\quad , h = \frac{100}{\pi r^2} = 5{,}031$ cm

b) $A = 2\pi r \cdot h + 4\pi r^2$

$A' = -\frac{200}{r^2} + 8\pi r$

$A' = 0 \Rightarrow r^3 = \frac{200}{8\pi} = \frac{25}{\pi}$

$r = \sqrt[3]{\frac{25}{\pi}} \cong \underline{\underline{2 \text{ cm}}} \quad , h = \underline{\underline{7{,}986}} \approx 8$ cm